Week 9: Exam-Day Strategy

Lecture notes · Thu Dec 3 / Fri Dec 4

The 2026 format (Saturday, December 5). Per the MAA, this year the competition runs as *four* 90-*minute sessions* with designated breaks between them, replacing the old pair of 3-hour sessions. Historically there are twelve problems, A1–A6 and B1–B6, worth 10 points each; this year they are distributed across the four sessions [SPLIT TBD: confirm the problem count and which problems fall in which session from the proctor's instructions]. Within each half the problems run roughly from easiest to hardest, so A1, A2, B1, B2 are the ones written to be solved. Once a session begins you may not leave and re-enter. Bring blue or black pens and your Putnam ID number from the registration platform; no calculators, computers, notes, books, or drawing tools, and phones and watches silenced and put away. Reed's exam room: **ROOM TBD**.

Triage: the first ten minutes of every session. Read every problem in the session before writing anything. For each, jot one line: what kind of object, what is asked, which week's toolbox it smells like (checklist below). Then start with the one you understand best, not the one with the smallest number. A1 and B1 are written to be solvable in 30–45 minutes; if nothing happens on one of them in 20 minutes you have probably misread it, so re-read word by word, try $n = 1, 2, 3$, and look at the equality or extremal case. Budget: in a 90-minute session, after 10 minutes of triage, roughly 35 minutes for your best problem, 25 for the next, and 15–20 minutes reserved to *write up* what you solved; a solved problem with no write-up is worth 0. Abandon a problem when you have had no new idea for 15 minutes and something easier is waiting (write the time of your last idea in the margin so you notice); return only after everything easier is written.

Writing for points. Graders give almost every paper 0, 1, 2 or 8, 9, 10: a proof is complete (with cosmetic slips) or it is not, and partial progress is worth a point or two at most. So aim for fewer problems, fully written. (1) State the answer first ("The answer is $n = 1$."), then prove it; "determine all" needs both directions, and the construction is usually the easy half. (2) Full sentences, every quantifier visible; a page of equations is not a proof. (3) The key step is exactly the one you must not hand-wave: if the proof turned on "the three valuations are pairwise distinct," write the line that shows it. (4) Name the tool as you use it ("by pigeonhole on the 12 vertices…") and cite standard theorems by name, but only ones you can state precisely. (5) Cross out abandoned work with one line; never leave two contradictory arguments for the grader to choose between. (6) "Evaluate" answers must be closed-form and *proved*; small cases are scratch work.

The night before and the morning of. Sleep is worth more than any theorem you could learn Friday night: stop by 9 pm, lay out pens and your Putnam ID number, go to bed. Eat a real breakfast; bring water and a quiet snack for the breaks. Be at **ROOM TBD** at least 20 minutes early so that seating, ID, and forms are not rushed. Between sessions walk, eat, and do not discuss the problems you could not solve; those scores are fixed and rehashing them drains the next session. The national median is usually 0 or 1: one complete solution is a genuine achievement.

Technique checklist: trigger phrases from Weeks 1–8.

- **Pigeonhole / extremal / induction (Wk 1).** "Among any N… some two," "show there exist," finite configurations: choose the boxes (residues, odd parts, partial sums, vertices) or take the largest, smallest, closest object. "For all n" with a smaller case inside: induction;

strengthen the statement if the step will not close.

- **Invariants and parity (Wk 2).** Moves, operations, "can one reach," "prove impossible": find a quantity that is preserved or monotone (parity, a sum mod m, a coloring, inversions).
- **Polynomials (Wk 3).** "Integer coefficients," roots, $P(a) - P(b)$: $(a - b) \mid P(a) - P(b)$, Vieta, compare degrees and leading terms, factor over $\mathbb{C}$, interpolate.
- **Number theory (Wk 4).** "Divisible by," "for all primes," nth powers: reduce mod a small prime, Fermat/Euler and orders, track v_p of each term (in $X + Y = Z$ the smallest valuation occurs twice), descent from a minimal solution.
- **Combinatorics (Wk 5).** "How many," "evaluate the sum": compute $k = 0, 1, 2, 3$ and guess, then bijection, double counting, recursion, or a generating function.
- **Inequalities and analysis (Wk 6).** "Show $\geq$," "largest M," limits, integrals: AM–GM, Cauchy–Schwarz, convexity; the equality case tells you the extremal example; telescope and squeeze.
- **Linear algebra (Wk 7).** Matrices, rank, orthogonal vectors: integer entries give an integer determinant and $\det(AA^{\mathsf{T}}) = (\det A)^2$; eigenvalues and trace; rank–nullity.
- **Probability and games (Wk 8).** "Random," "expected," "two players": linearity of expectation, symmetry (swap heads and tails), pairing and strategy stealing, work backwards from small positions.

Pitfall. Three ways to lose 8 points on a solved problem: proving one direction of a "determine all"; using a remembered fact without checking its hypotheses (AM–GM needs nonnegative numbers, "the minimum exists" needs a finite set); running out of time before the key step is on paper. Three ways to lose a session: 60 minutes on A4 because it looked interesting, refusing to re-read a statement, not eating.

Suggested timing (15 minutes). 0–3: format, what to bring, room, arrival time. 3–7: triage and the time budget. 7–11: writing for points and scoring; show "answer first, both directions" on a determine-all problem. 11–15: walk down the checklist, then start the 35-minute timed set (Problems 1–3, alone, hard stop).