Math 111: Calculus   Homework 1

Due Friday, September 18 by 10:00 P.M. via Gradescope

Covers Active Calculus §1.1 and §1.3–§1.5.

Before you start. Write in sentences and paragraphs, as an explanation another student in this course could read and follow. Draw diagrams large and label them. You are encouraged to work with classmates; cite everyone you worked with, and write your solutions independently — duplicated solutions are an Honor Principle violation. Each problem is marked holistically on the 0–5 scale, with up to two further points for the quality of the writing. If you need an extension, use the extension form *before* the deadline.

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1. A cup of coffee, and how well you can know its rate

A cup of coffee is left on a desk. Its temperature T, in degrees Fahrenheit, is recorded every two minutes:

t (minutes)	0	2	4	6	8	10
$T(t)$ (°F)	200	185	173	163	155	149

There is no formula for T. The table is all you have.

(a) Using the table, produce one estimate of $T'(4)$ that you believe is too large and one that you believe is too small. Give units, and say which is which *and how you know.*

(b) State your best estimate of $T'(4)$, together with a bound on how far off it could be. Your bound must be one you can defend from the data — say exactly what you are assuming about how T' behaves between $t = 2$ and $t = 6$ in order to defend it.

(c) What additional measurement would let you cut your bound roughly in half? Be specific about when it would be taken.

(d) From the table alone, is T' increasing or decreasing over these ten minutes? Say what your answer means for someone waiting to drink the coffee.

2. Bicycles and degrees

Let $B(h)$ be the number of bicycles that cross the Hawthorne Bridge on a day whose high temperature is h degrees Fahrenheit. Suppose $B(60) = 4300$ and $B'(60) = 210$.

(a) Explain, in one or two sentences a reader who has never taken calculus could follow, what $B'(60) = 210$ says about bicycles and weather. State the units of $B'(60)$ and say where they come from.

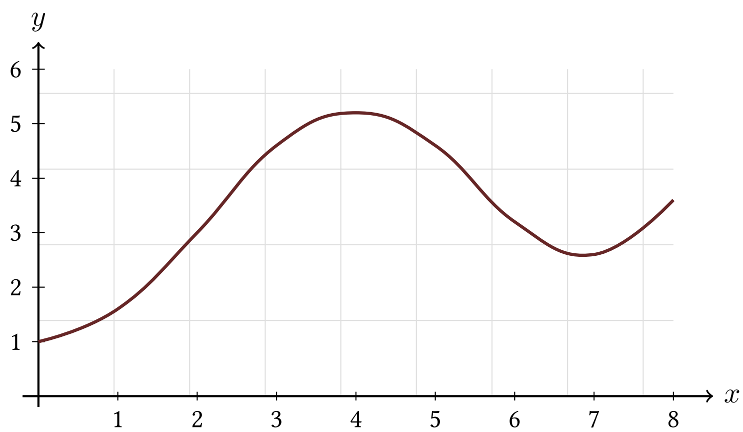
(b) Would you expect $B'(95)$ to be positive or negative? Explain your reasoning in terms of the situation, not in terms of formulas.

(c) A classmate writes: "$B'(60) = 210$ means that 210 more bicycles cross on a 61-degree day than on a 60-degree day." This is not quite right. Explain what is wrong with it, write down a corrected version, and say under what circumstances the classmate's sentence would be a good approximation.

(d) Suppose you also learn that B' is decreasing for temperatures near 60 degrees. What does that tell you about bicycle traffic near 60 degrees? Answer in a sentence about bridges and weather.

3. Reading a derivative off a picture

The graph of a function f on $[0, 8]$ is shown below. No formula for f is given, and none is needed.



(a) On axes of your own, sketch the graph of f' on $[0, 8]$. Label the scale on your vertical axis, and make clear where f' is zero, positive, and negative.

(b) Put $f'(1)$, $f'(4)$, and $f'(6)$ in increasing order. Justify the ordering from the picture of f, without reading values off your sketch of f'.

(c) On which subintervals of $[0, 8]$ is f' increasing? Justify your answer using the shape of the graph of f itself, not by reading your sketch from part (a).

(d) Which of the following can you determine from this picture, and which can you not: $f(3)$, $f'(3)$, and the value of x at which f' is largest? For each, either give a value or a range you can defend, or explain what makes it undeterminable.

4. Making examples, and finding the broken step

(a) Sketch the graph of a single function g on $[0, 4]$ satisfying all of the following at once: $g(1) = 3$; $g'(1) = 0$; $g'(x) > 0$ for $0 < x < 1$; $g'(x) < 0$ for $1 < x < 3$; and g' is increasing for $2 < x < 4$. Label the features that show each condition is met.

(b) Give a function h with $h'(2) = 0$ whose graph has neither a peak nor a valley at $x = 2$. A formula or a carefully drawn graph is fine. Explain how you know your example works.

(c) A student submits the following argument.

> **Claim.** If f is increasing on an interval, then f' is increasing on that interval.
> **Argument.** Suppose f is increasing. Then $f(b) > f(a)$ whenever $b > a$ in the interval. So the difference quotient $\dfrac{f(b) - f(a)}{b - a}$ is positive for every such pair. Taking b close to a, it follows that $f'(a)$ is positive at every point of the interval. A positive quantity gets larger as x increases, so f' is increasing.

Identify the *first* sentence of the argument that is wrong, and say precisely what is wrong with it. Then give a specific function showing the claim itself is false. Finally, state a correct claim of the same shape — something true that the first part of this argument does establish — and say why it is true.