24. X. 14

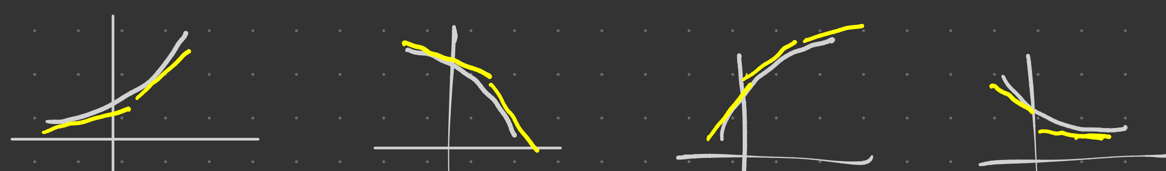
Goals

- First derivative test
- Concavity & inflection points
- Second derivative test

Recall Cor 3 of MVT:

$f' > 0$ on $I \Rightarrow f$ increasing on I

$f' < 0$ on $I \Rightarrow f$ decreasing on I



$f' > 0$
f increasing
note: f' also increasing

$f' < 0$ dec
f dec

$f' > 0$ but dec
still f increasing

$f' < 0$ inc
f decreasing

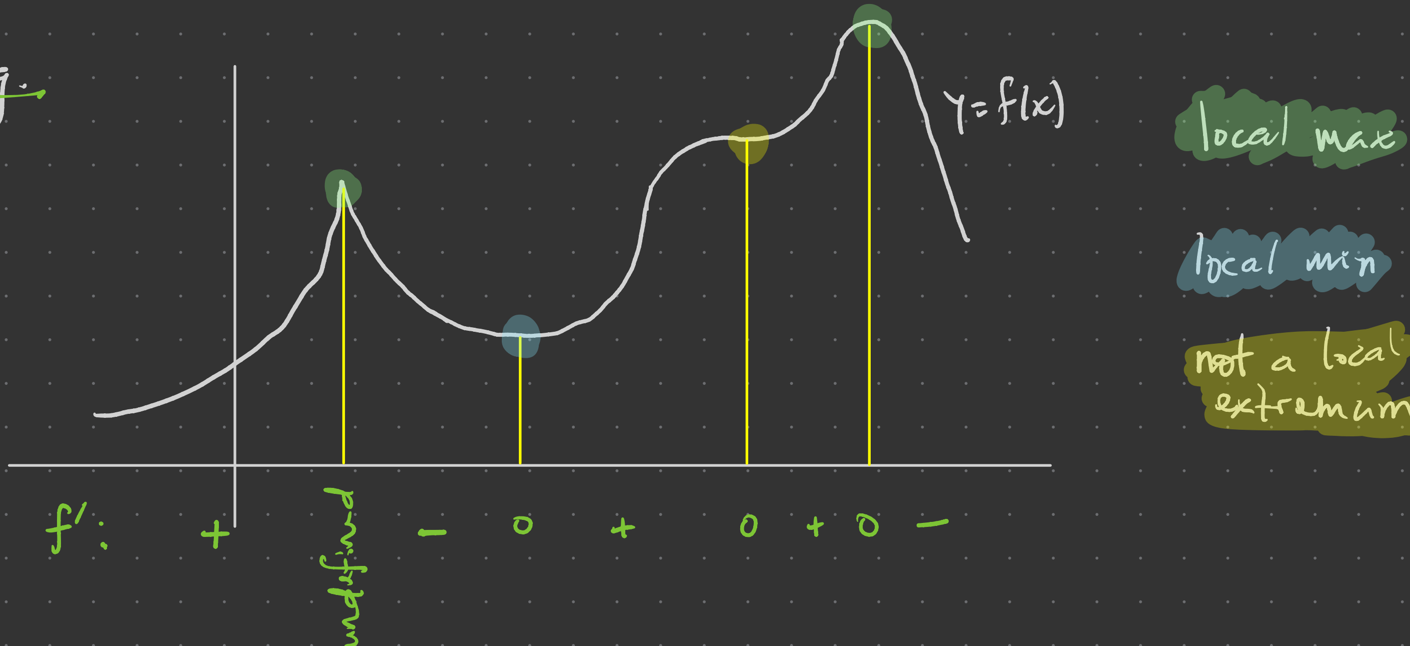
First derivative test Suppose f cts over interval I containing a critical point c. If f is diff'l over I except possibly at c, then $f(c)$ satisfies one of the following descriptions:

(i) if f' changes sign from positive (for $x<c$) to negative (for $x>c$), then $f(c)$ is a local max of f

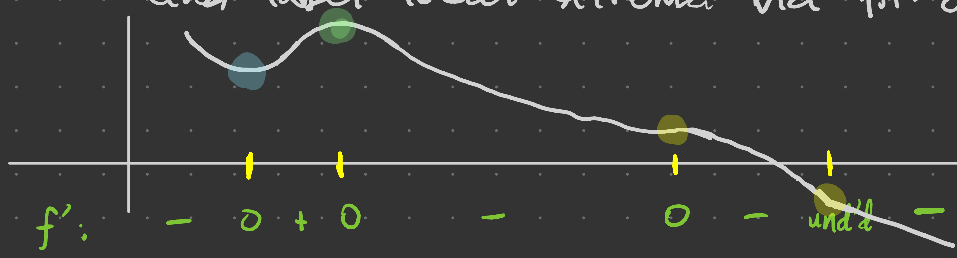
(ii) if f' changes sign from negative (for $x<c$) to positive (for $x>c$), then $f(c)$ is a local min of f

(iii) if f' has the same sign left and right of c, then $f(c)$ is not a local extremum.

E.g.



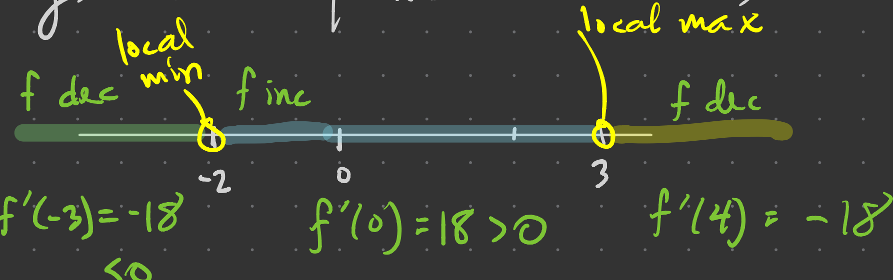
Problem Draw a graph with the following "signature": and label local extrema via 1st deriv test



E.g. Let's use the first derivative test to find all local extrema of $f(x) = -x^3 + \frac{3}{2}x^2 + 18x$.

We have
$$
\begin{aligned}
f'(x) &= -3x^2 + 3x + 18 \\
&= -3(x^2 - x - 6) \\
&= -3(x-3)(x+2)
\end{aligned}
$$

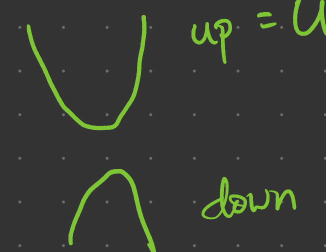
so the only critical points are $x = -2, 3$.



By the 1st deriv test, $f(-2) = \_\_\_$ is a local min & $f(3) = \_\_\_$ is a local max.

Concavity

Defn Let f be a function that is diff'l on an open interval I.
If f' is increasing over I, we say f is concave up on I;
if f' is decreasing over I, we say f is concave down on I.



E.g.

f inc, f' inc $\Rightarrow$ conc up

f inc, f' dec $\Rightarrow$ conc down

f dec, f' inc $\Rightarrow$ conc up

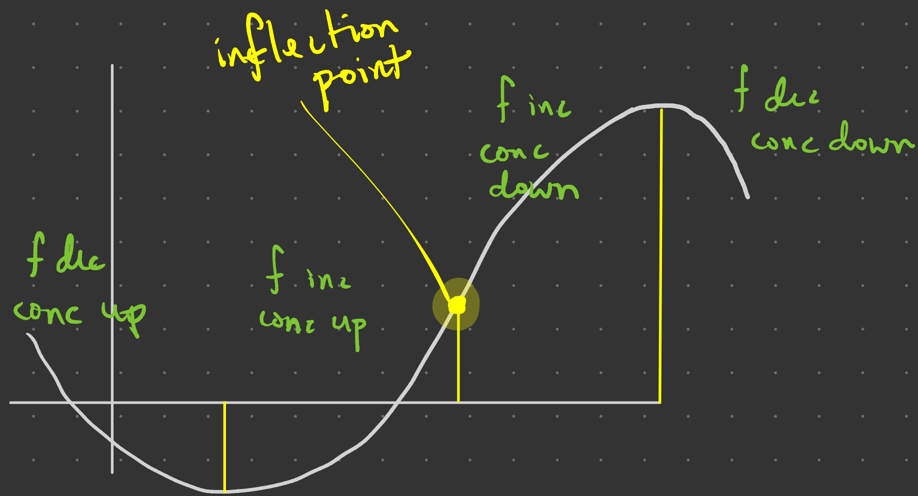
f dec, f' dec $\Rightarrow$ conc down

Concavity test Let f be twice diff'l on an interval I.

(i) If $f''(x) > 0$ for all $x \in I$, then f is concave up on I.

(ii) If $f''(x) < 0$ for all $x \in I$, then f is concave down on I.

Defn If f is cts at a and f changes concavity at a, then point $(a, f(a))$ is an **inflection point** of f



$f' < 0$	$f' > 0$	$f' > 0$	$f' < 0$
$f'' > 0$	$f'' > 0$	$f'' < 0$	$f'' < 0$

inflection point noun

1 : a moment when significant change occurs or may occur : TURNING POINT

At 18, Bobby is at an *inflection point* that will largely determine the course of his life.
— Stacy Perman

... the gradual move away from big iron machines toward work stations and personal computers has been going on for years in corporate America—but the *inflection point* came suddenly.
— Steve Lohr

It depends on us, on the choices we make, particularly at certain *inflection points* in history; particularly when big changes are happening and everything seems up for grabs.
— Barack Obama

2 mathematics : a point on a curve that separates an arc concave upward from one concave downward and vice versa

Second derivative test

Suppose $f'(c)=0$, f'' is continuous over an interval containing c.

(i) If $f''(c)>0$, then f has a local min at c

(ii) If $f''(c)<0$, then f has a local max at c

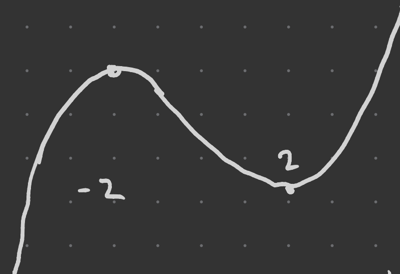
(iii) If $f''(c)=0$, then the test is inconclusive.

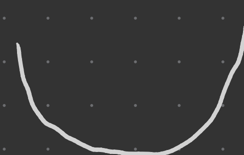
E.g. If $f(x)=x^3-12x+5$ then $f'(x)=3x^2-12=3(x^2-4)=3(x+2)(x-2)$.

Thus $f'(x)=0$ for $x=\pm 2$. Now $f''(x)=6x$ and

$f''(-2)=-12 \implies x=-2$ is a local max

$f''(2)=12 \implies x=2$ is a local min.





E.g.

$f(x) = x^4$

$f'(x) = 4x^3 = 0$ iff $x = 0$

$f''(x) = 12x^2 \Rightarrow f''(0) = 0$ second deriv test inconclusive

Multivariable

saddle