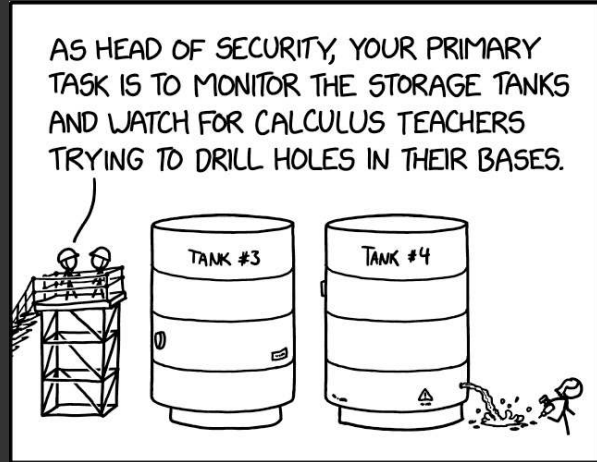


Related Rates

Idea: Implicit differentiation relates the rates of change of quantities appearing together in an equation. We can use this to solve cool problems.

E.g. Spherical balloon inflating at a constant rate of $2 \text{ cm}^3/\text{sec}$.
How fast is the radius increasing when the radius is 6 cm ?

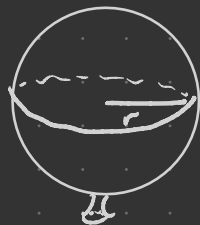


xkcd 2974

Know $V = \frac{4}{3}\pi r^3$, with both V and r functions of time t .

Differentiating with respect to t :

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$



We are told $\frac{dV}{dt} = 2 \text{ cm}^3/\text{sec}$, so

$$\frac{dV}{dt} = 2 \text{ cm}^3/\text{sec}$$

$$2 = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{2\pi r^2}$$

$$\text{Thus } \left. \frac{dr}{dt} \right|_{r=6 \text{ cm}} = \frac{1}{2\pi \cdot 6^2} \text{ cm/sec} = \frac{1}{72\pi} \text{ cm/sec} \quad \checkmark$$

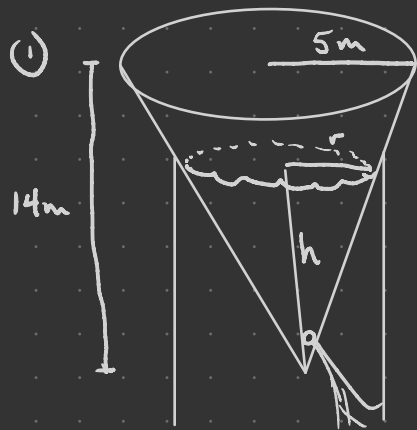
General Strategy

- ① Draw and label
- ② State given info + rate to be determined
- ③ Find an equation relating the variables
- ④ Implicitly differentiate
- ⑤ Substitute known info from ② and solve for desired rate — include units!

E.g. A conical tank of water has a height of 14 meters and radius of 5 meters at the top. Your calculus teacher has drilled a hole into the bottom of the tank

and it is leaking water at a rate of $10 \text{ cm}^3/\text{sec}$.

How quickly is the height of the water in the tank changing when its height is 5 meters?



② Know $\frac{dV}{dt} = -10 \text{ cm}^3/\text{sec}$

Want $\frac{dh}{dt} \Big|_{h=5\text{m}}$

neg b/c vol decreasing

③ $V = \frac{1}{3} \pi r^2 h$

is the volume of water

By similar triangles, $\frac{5}{14} = \frac{r}{h} \Rightarrow r = \frac{5h}{14} \text{ m}$

and $V = \frac{1}{3} \pi \left(\frac{5}{14}\right)^2 h^2 h = \frac{5\pi}{588} h^3$

$$(4) \quad \frac{dV}{dt} = \frac{5\pi}{588} 3h^2 \frac{dh}{dt} = \frac{15\pi}{588} h^2 \frac{dh}{dt}$$

$$(5) \quad \text{Know } \frac{dV}{dt} = -10 \text{ cm}^3/\text{sec} = -10 \cdot \left(\frac{1}{100} \frac{\text{m}}{\text{cm}}\right)^3 \cdot \frac{\text{cm}^3}{\text{sec}} \\ = -10^{-5} \frac{\text{m}^3}{\text{sec}}$$

$$\text{So } -10^{-5} = \frac{15\pi}{588} h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-588}{15\pi \cdot 10^5 h^2}$$

$$\text{and } \left. \frac{dh}{dt} \right|_{h=5 \text{ m}} = \frac{-588}{15\pi \cdot 10^5 \cdot 25} \text{ m/sec} \approx -4.99 \cdot 10^{-6} \text{ m/sec}$$

Problem 1



total resistance R satisfies

Ohm's law $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Suppose R_1 increasing at a rate of 0.4 Ohms/sec

R_2 decreasing at a rate of 0.5 Ohms/sec

At what rate is R changing when $R_1 = 100 \text{ Ohms}$, $R_2 = 111 \text{ Ohms}$?

Problem 2

Person A standing at $(0,0)$ begins walking north at a rate of 20 units/sec ; person B standing at $(0,50)$ begins walking west at a rate of 10 units/sec . How quickly is the distance between them changing when they are 70 units apart?

Problem 3 Boyle's law says $PV = c$ in a gas

with constant temperature where P = pressure, V = volume, and c is a constant.

Suppose a gas is in a cylinder with piston and its initial volume is 250 cm^3 , pressure 100 kPa . The piston is depressed so that volume decreases at a rate of $50 \text{ cm}^3/\text{min}$. How quickly will the pressure of the gas initially increase?

