

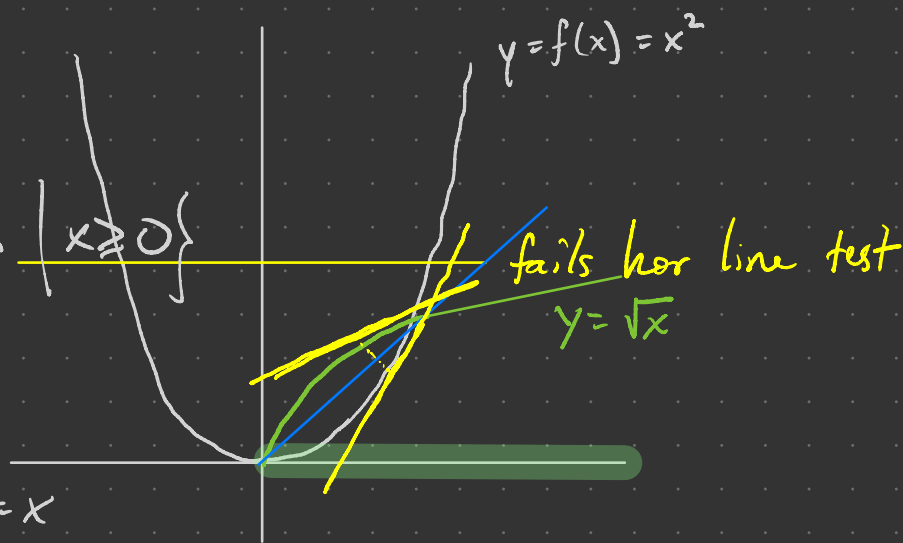
E.g. $f(x) = x^2$

Restrict the domain
of f to $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \mid x \geq 0\}$

$g(x) = \sqrt{x}$ on $\mathbb{R}_{\geq 0}$.

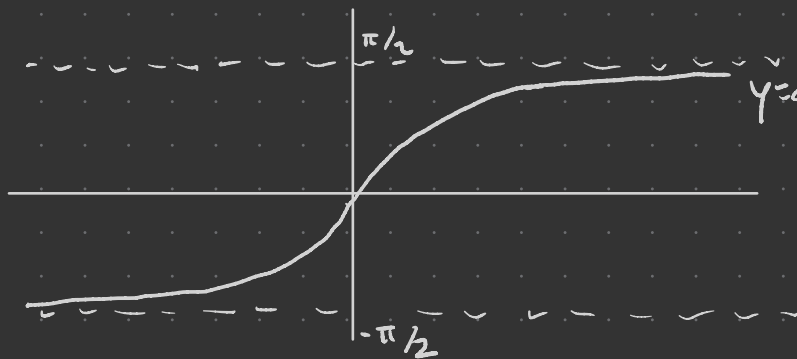
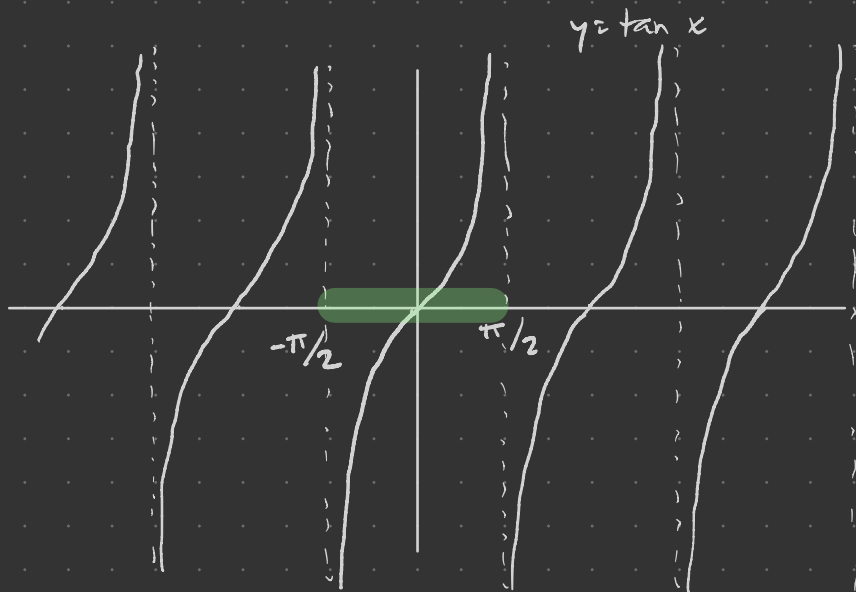
Then $f(g(x)) = (\sqrt{x})^2 = x$

$g(f(x)) = \sqrt{x^2} = x$



Note Graph of $y = f^{-1}(x)$ is the reflection of $y = f(x)$ over $y = x$ line.

E.g. $f(x) = \tan x$



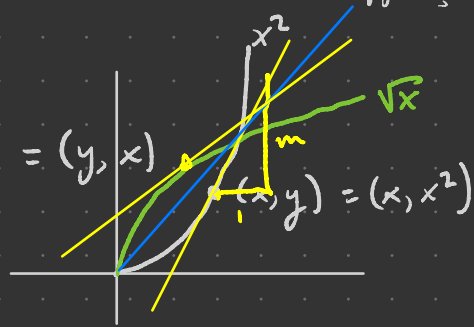
Write $\arctan$ for $\tan^{-1}$. Note need to choose region where hor line test hold

Inverse Function Theorem Suppose f is both invertible and diff'1.

Then for all x such that $f'(f^{-1}(x)) \neq 0$,

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(x^2, x) = (y, x)$$



Pf We know $x = f(f^{-1}(x))$. Differentiating both sides,

$$1 = f'(f^{-1}(x)) \cdot (f^{-1})'(x)$$

$$\Rightarrow (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \quad [\text{divide by } f'(f^{-1}(x))]$$

E.g. If $f(x) = x^2$ then for $x \geq 0$, $f^{-1}(x) = \sqrt{x}$.

$$\text{Thus } (\sqrt{x})' = \frac{1}{f'(\sqrt{x})} = \frac{1}{2\sqrt{x}}.$$

$$\text{Similarly, for all } x, \quad \underbrace{(\sqrt[3]{x})'}_{x^{1/3}} = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3} x^{-2/3}.$$

In fact, this is how we extend the power rule to all rational #'s!

$$\text{i.e. } (x^{1/3})' = \frac{1}{3} x^{1/3 - 1} = \frac{1}{3} x^{-2/3}$$

Q What about irrational exponents? $\frac{d}{dx}(x^\pi) = \pi x^{\pi-1}$ ✓

Rate of change

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \text{instantaneous rate of change of } f \text{ at } x$$

If $s(t)$ = position along a line at time t

then $s'(t) = v(t)$ = velocity at time t , $|v(t)|$ = speed $\left\{ \begin{array}{l} v = \frac{ds}{dt} \end{array} \right.$

$s''(t) = v'(t) = a(t)$ = acceleration at time t $\left\{ \begin{array}{l} a = \frac{dv}{dt} \end{array} \right.$

Units : • If $s(t)$ in meters, t in seconds, then $\frac{s(t+h) - s(t)}{h} = \frac{d}{dt} \left(\frac{ds}{dt} \right) = \frac{d^2 s}{dt^2}$
 in $\frac{\text{meters}}{\text{second}} = \frac{\text{m}}{\text{s}}$ and so is $v(t)$.

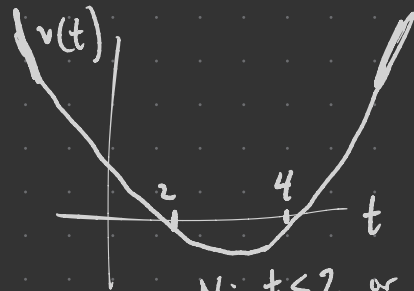
• Then $a(t)$ is the limit of $\frac{v(t+h)-v(t)}{h}$ which is in $\frac{m/s}{s} = \frac{m}{s^2} = \frac{\text{meters}}{\text{second}^2}$

Problem Position $s(t) = t^3 - 9t^2 + 24t + 4$ meters north of current location t seconds from now.

(a) Find $v(t)$

(b) When is the object at rest?

(c) When is it moving north? south?



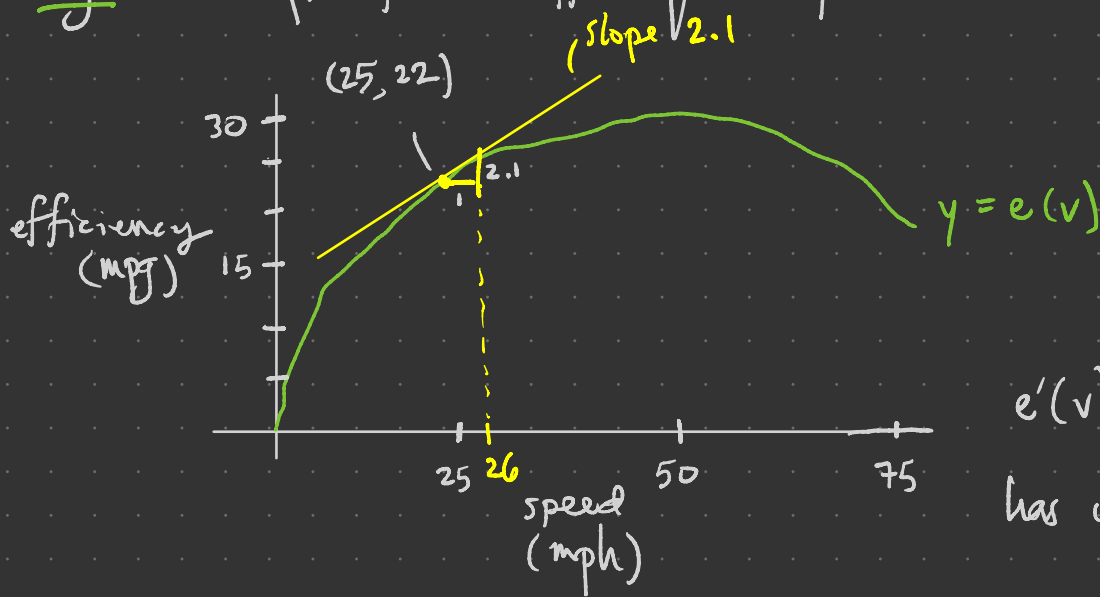
$$(a) \quad v(t) = s'(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$$

N: $t < 2$ or $t > 4$
S: $2 < t < 4$

$$(b) \quad v(t) = 0 \iff 0 = 3(t^2 - 6t + 8) = 3(t-2)(t-4)$$

so at rest for $t = 2$ or 4 seconds

Ex. Graph of fuel efficiency vs speed:



$$e'(v) = \lim_{h \rightarrow 0} \frac{e(v+h) - e(v)}{h}$$

has units $\frac{\text{mpg}}{\text{mph}}$

$$= \frac{\text{mi/gal}}{\text{mi/hr}}$$

$$= \text{hr/gal}$$

Interpret $e'(25) = 2.1 \text{ hr/gal}$:

If we increase speed from 25 to 26,
change in efficiency is ≈ 2.1 .

Restrict e to $[0, 50]$ and let $f = e^{-1}$. What is $f'(22)$ and what does it mean?

$$f'(22) = \frac{1}{\underbrace{e'(25)}_{f(22)}} = \frac{1}{2.1} \approx 0.476$$

$$\left. \begin{array}{l} \frac{\text{mpg}}{\text{mph}} = \frac{\text{mi/gal}}{\text{mi/hr}} \\ = \text{hr/gal} \end{array} \right\}$$

Note $f(\text{efficiency})$

= speed b/w 0 and 50
with that assoc
efficiency