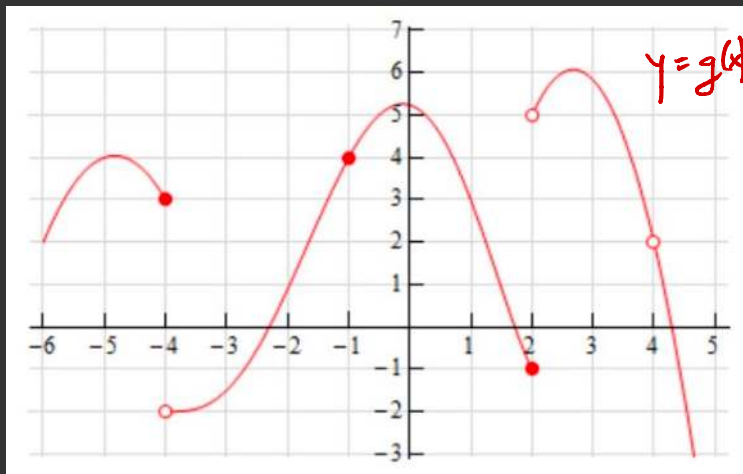


Quiz

Compute $\lim_{x \rightarrow a} g(x)$ for $a = -4, -1, 2, 4$.

$$\lim_{x \rightarrow a} g(x) \quad \left| \begin{array}{c|c|c|c} \text{DNE} & 4 & \text{DNE} & 2 \end{array} \right.$$

$$\lim_{x \rightarrow a^+} g(x) \quad \left| \begin{array}{c|c|c|c} -2 & 4 & 5 & 2 \end{array} \right.$$

$$\lim_{x \rightarrow a^-} g(x) \quad \left| \begin{array}{c|c|c|c} 3 & 4 & -1 & 2 \end{array} \right.$$

Some functions only have a limit from one "side".

$$\lim_{x \rightarrow a^+} f(x)$$

x approaches
 a from the right

$$\lim_{x \rightarrow a^-} f(x)$$

x approaches
 a from the left

(See book for definitions.) Problem What about left and right variants of limits above?

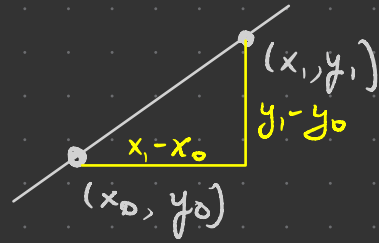
Problem (a) Graph $y = x^2$

(b) Use the following table to compute slopes of lines through $(1, 1)$ and (x, x^2)

x	x^2	$\frac{x^2-1}{x-1}$
1.1	1.21	2.1
1.01	1.0201	2.01
1.001	1.002001	2.001
0.999	0.998001	1.999
0.99	0.9801	1.99
0.9	0.81	1.9

slope of line
through (x_0, y_0)
and (x_1, y_1) is

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{\text{rise}}{\text{run}}$$



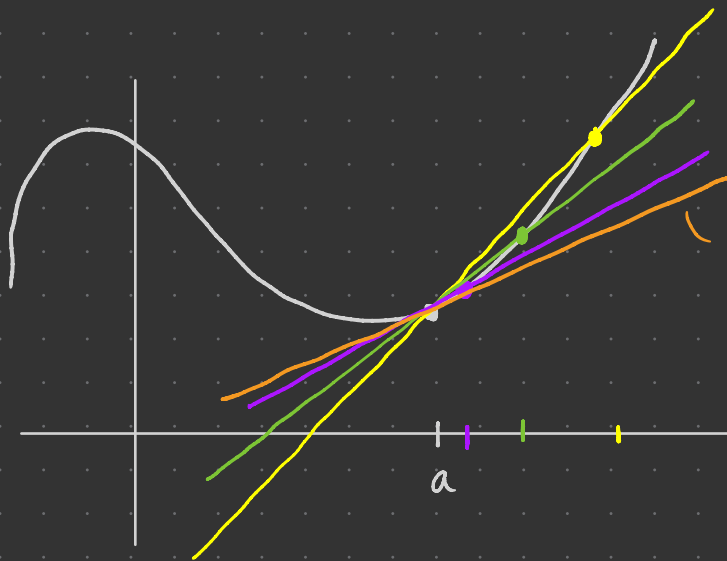
(c) What is the limit of the secant
slopes as $x \rightarrow 1$?

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$$

or set $m(x) = \frac{x^2 - 1}{x - 1}$ then $\lim_{x \rightarrow 1} m(x) = 2$

You just computed your first derivative!

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$



slope $f'(a)$ = instantaneous
rate of change
of $f(x)$ as
 $x \rightarrow a$.

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} =$$

$$\lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1}$$

$$= \lim_{x \rightarrow 1} (x+1)$$

$$= 1+1 = 2$$

Discussion What is the relationship between

$$\lim_{x \rightarrow a} f(x), \quad \lim_{x \rightarrow a^+} f(x), \quad \text{and} \quad \lim_{x \rightarrow a^-} f(x) ?$$

In particular, what does $\lim_{x \rightarrow a} f(x) = L$ tell you about

$$\lim_{x \rightarrow a^+} f(x), \quad \lim_{x \rightarrow a^-} f(x) ? \quad \text{What does } \lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x)$$

tell you about $\lim_{x \rightarrow a} f(x) ?$

both equal L !

then $\lim_{x \rightarrow a} f(x)$
does not exist

Also, if $\lim_{x \rightarrow a^+} f(x) = L = \lim_{x \rightarrow a^-} f(x)$, then $\lim_{x \rightarrow a} f(x) = L$.
(In fact, equivalent!)

The sign function is $\text{sign} : \mathbb{R}_{\neq 0} \rightarrow \mathbb{R}$ given by

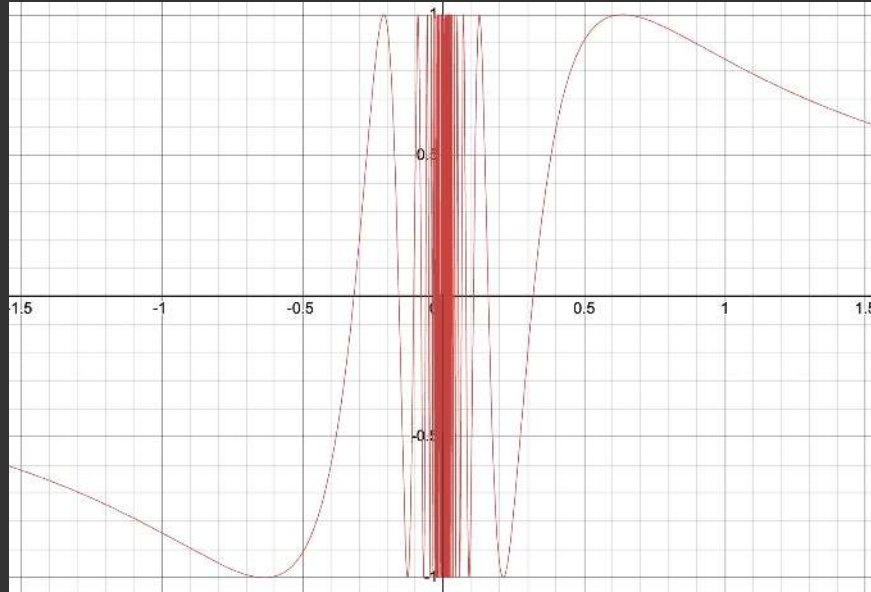
$$\text{sign}(x) = \frac{x}{|x|} \text{ for } x \neq 0.$$

(a) Graph $\text{sign}(x)$

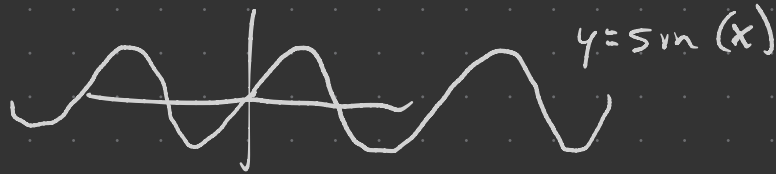
(b) Compute $\lim_{x \rightarrow 0^+} \text{sign}(x)$, $\lim_{x \rightarrow 0^-} \text{sign}(x)$

(c) Fix a in $\mathbb{R}$. Where does $\text{sign}(x-a)$ fail to have a limit?

What about $\lim_{x \rightarrow 0^+} \sin\left(\frac{1}{x}\right)$?

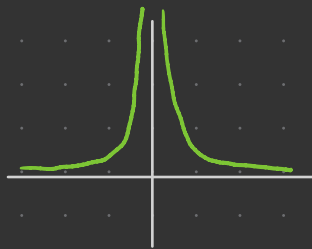


$$y = \sin\left(\frac{1}{x}\right)$$



Infinite limits, limits at infinity

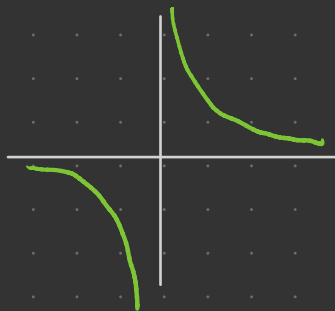
Consider $f(x) = \frac{1}{x^2}$:



Say $\lim_{x \rightarrow 0} f(x) = +\infty$ b/c as x gets close to 0 (from either side), $f(x)$ gets arbitrarily large.

What about $\lim_{x \rightarrow 0} \frac{1}{x}$?

DNE



Say $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ because as x gets arbitrarily large,

$\frac{1}{x}$ gets closer and closer to 0.