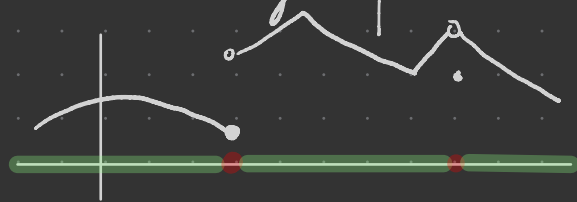


Continuity A function f is continuous at $x=a$ when

$$\lim_{x \rightarrow a} f(x) = f(a)$$

A function f is continuous on an open interval when it is continuous at every point in that interval.



cts

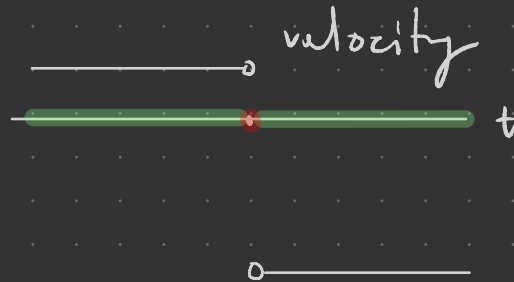
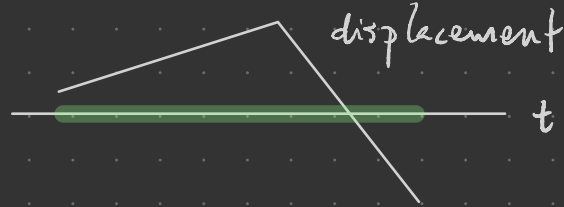
dis cts

E.g.

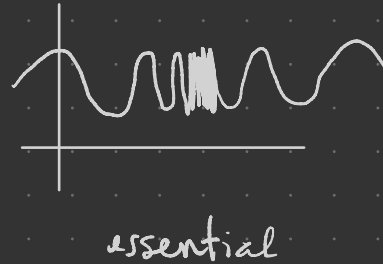
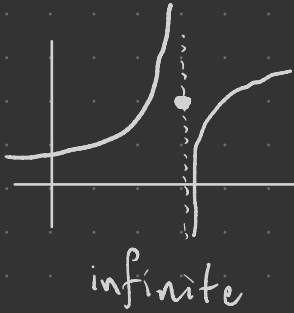
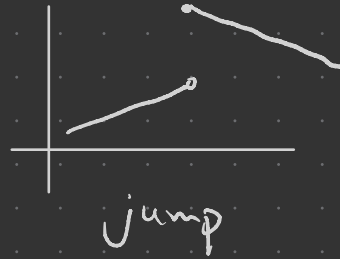
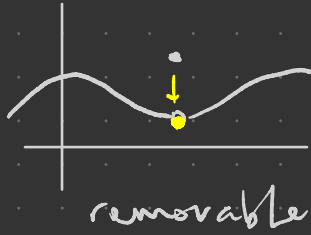
- Polynomial functions are continuous on $\mathbb{R}$
- Rational functions are continuous on their domains
- Trig functions are continuous on their domains

- Composites of cts functions are cts.

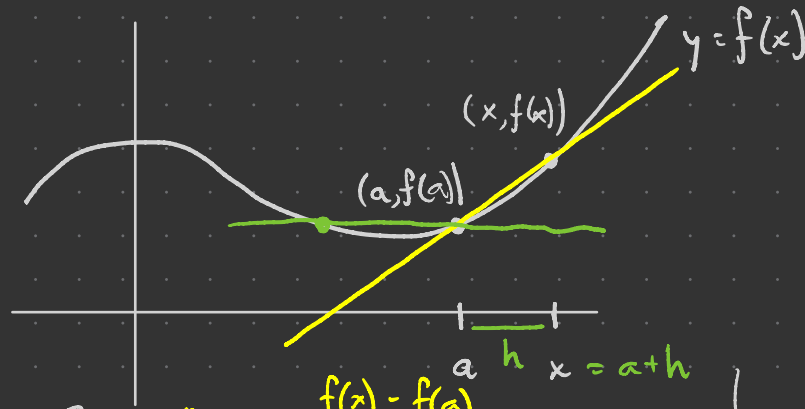
E.g. A fly travels east at a constant speed until it impacts a westbound freight train:



Types of discontinuity



Secants



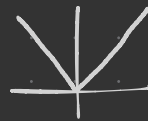
slope of
secant

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}$$

$$= \frac{f(a+h) - f(a)}{h}$$

$\lim_{x \rightarrow a}$ allows
 $x < a$

$\lim_{h \rightarrow 0}$ allows $h < 0$



difference
quotients

$$\lim_{x \rightarrow 0} \frac{|x| - |0|}{x - 0}$$

DNE

Tangents $x \rightarrow a$ (or $h \rightarrow 0$)

$$m_{\text{tan}} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

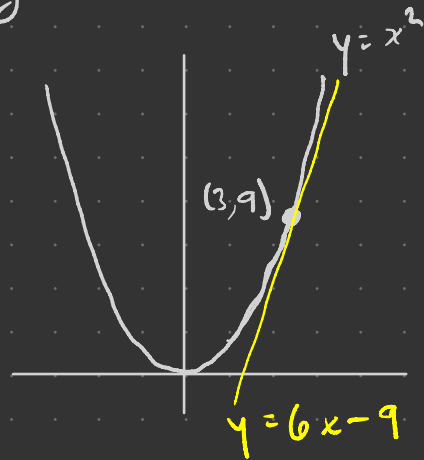
E.g. Let's find the tangent line to $f(x) = x^2$ at $x=3$:

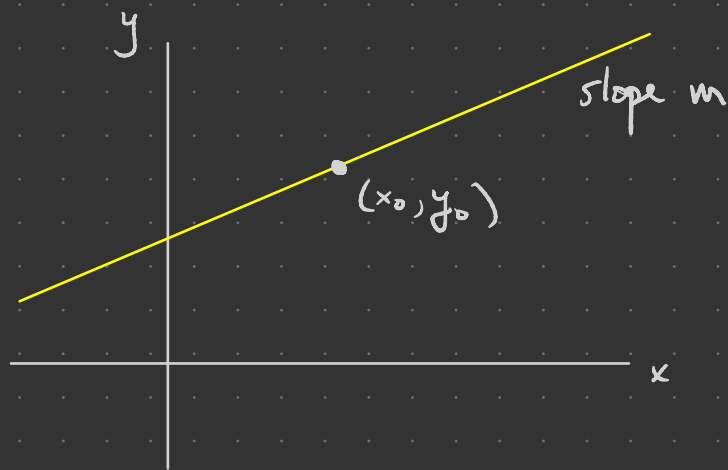
$$\begin{aligned} m_{\text{tan}} &= \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x-3} \\ &= \lim_{x \rightarrow 3} (x+3) \\ &= 6 \end{aligned}$$

The tangent line passes through $(3, 9)$ with slope 6, so

$$y - 9 = 6(x - 3)$$

$$\Leftrightarrow y = 6x - 9$$





point-slope: line has eq'n

$$y - y_0 = m(x - x_0)$$

$$(x_0, y_0) = (3, 9) \quad m = m_{\tan} = 6$$

Note Also have

$$m_{\text{tan}} = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$$

$$= \lim_{h \rightarrow 0} \frac{9 + 6h + h^2 - 9}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} (6+h)$$

$$= 6 \text{ as before } \checkmark$$

Problem Find $m_{\tan}$ at $x=4$ for $f(x)=\sqrt{x}$ via one of limit def'n's.

$$m_{\tan} = \lim_{x \rightarrow 4} \frac{\sqrt{x} - \sqrt{4}}{x - 4}$$

$x - 4 = (\sqrt{x} - 2) \cdot (\sqrt{x} + 2)$

$$= \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}$$

$$= \lim_{x \rightarrow 4} \frac{\cancel{x} - 4}{(\cancel{x} - 4)(\sqrt{x} + 2)} = \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{4}$$

Is $\sqrt{1} + \sqrt{4} = \sqrt{5}$?
Then is $\sqrt{x} + \sqrt{y} = \sqrt{x+y}$?

NO!

Derivatives

Defn Let $f(x)$ be a function defined on an open interval containing a . The derivative of $f(x)$ at a is

$$f'(a) := \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

$$\text{I.e., } f'(a) = m_{\text{tan}} !$$

Rate of change

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a} = \frac{\Delta f}{\Delta x} = \text{average rate of change over } [a, x]$$

change in f (pointing to Δf)
change in $x = h$ (pointing to Δx)

So $m_{\text{tan}} = f'(a) = \text{instantaneous rate of change}$

If $f(t)$ is displacement or position at time t ,
then $f'(t)$ is velocity